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Finite time thermodynamic evaluation of irreversible Ericsson and Stirling heat engines

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Abstract

This paper presents an investigation on finite time thermodynamic evaluation of Ericsson and Stirling heat engines. Finite time thermodynamics has been applied to optimise the power output and thermal efficiency of both engines, including finite heat capacitance rates of the heat source and sink reservoirs external fluids, finite time heat transfer, regenerative heat losses and direct heat leak losses from source to sink reservoirs. The presence of these effects causes irreversibilities and affects the performance of the Ericsson and Stirling heat engines using ideal or real gas as the working fluid/substance. It is found that both cycles with an ideal regenerator ($\epsilon_R = 1.00$) are as efficient as an endoreversible Carnot heat engine operating at the same conditions, but this is not practical because an ideal regenerator requires either infinite regenerator area or infinite regeneration time. It is also found that the regenerative heat losses, regenerator effectiveness and the direct heat leak losses do not affect the maximum power output of either heat engine. Some special cases are also described. © 2000 Elsevier Science Ltd. All rights reserved.

Keywords: Ericsson and Stirling heat engine; Finite time thermodynamics; Regenerative heat loss; Direct heat leak loss; Finite time heat transfer; Irreversible process

1. Introduction

Finite time thermodynamics came into existence after the novel work of Curzon–Ahlborn

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Nomenclature

A_H heat transfer surface area between heat source and heat engine (m^2)
A_L heat transfer surface area between heat source and heat sink (m^2)
C_H heat capacitance rate of heat source reservoir fluid (kW/K)
C_L heat capacitance rate of heat sink reservoir fluid (kW/K)
n mole numbers of the heat engine working fluid
P power output (kW)
p_1 pressures of working fluid during process 4-1 (kPa)
p_2 pressure of working fluid during process 2-3 (kPa)
P_m maximum power output (kW)
Q_h heat absorbed by working fluid during process 3-4 (kJ)
Q_C heat released by working fluid during process 1-2 (kJ)
Q_H amount of total heat absorbed from heat source (kJ)
Q_L amount of total heat released to heat sink (kJ)
Q_R regenerative heat transfer to heat sink (kJ)
ΔQ heat loss during regenerative process per cycle (kJ)
T_{H1} initial temperature of heat source (K)
T_{H2} final temperature of heat source (K)
T_{L1} initial temperature of heat sink (K)
T_{L2} final temperature of the heat sink (K)
T_h temperature of working fluid during isothermal heat addition process 3-4 (K)
T_C temperature of working fluid during isothermal heat rejection process 1-2 (K)
t_{cycle} total cycle time (s)
t_H time of heat addition process 3-4 (s)
t_L time of heat rejection process 1-2 (s)
t_R time of regenerative heat addition/rejection process (s)
U_H overall heat transfer coefficient between heat source and heat engine (W/K m^2)
U_L overall heat transfer coefficient between heat engine and heat sink (W/K m^2)
v_1 volume of working fluid during process 2-3 (m^3)
v_2 volume of working fluid during process 4-1 (m^3)
η thermal efficiency of heat engine
η_m thermal efficiency at maximum power output
η_{CA} Curzon–Ahlborn efficiency
η_I 1st law efficiency
η_{II} 2nd law efficiency
ϵ_H effectiveness of high temperature heat exchanger
ϵ_L effectiveness of low temperature heat exchanger
ϵ_R effectiveness of regenerator

[1]. Since then, it has been applied on various heat engine and refrigeration cycles. A distinction between Ericsson and Stirling cycle machines has not been well presented in the literature of both, with the meaning of the latter often being understood as a closed cycle version of the former. However, in thermodynamic terms, the Ericsson cycle is considered to be either a closed or an open cycle. In both cycles, the primary heat addition and rejection processes can be modelled as occurring at constant temperatures, while the regeneration processes can be modelled as isobaric (in the Ericsson cycle) or isochoric (in the Stirling cycle) processes.

Ericsson and Stirling engines have captivated several generations of engineers and physicists due to their potential to provide a high conversion efficiency. However, use of these engines did not prove to be successful due to the relatively poor material technologies available at that time. As the world community has became much more environmentally conscious, further attention to these engines has been received because these engines are inherently clean and thermally more efficient. Moreover, as a result of advances in material technology, these engines are currently being considered for a variety of applications [2–5]. These engines are also under research and development for their use as heat pumps, replacing systems that are not eco-friendly and environmentally acceptable [6,7]. Today, however, the popularity of these engines is growing rapidly due to their many advantages, like low noise, less pollution levels and their flexibility as external combustion engines to utilise a variety of energy sources, all having added to the momentum of their rise [8–12]. As a result, these engines are currently being used or are being considered for use in a wide variety of applications [12–21], including both terrestrial [16–18] and non-terrestrial [12–15] solar installations.

In this paper, we have presented a general and combined analysis on the finite time thermodynamic evaluation of Ericsson and Stirling heat engines, including finite time heat transfer, finite heat capacitance rates of external fluids in the heat source and sink reservoirs, regenerative heat losses, finite effectivenesses of the hot and cold side heat exchangers and direct heat leak losses, and obtained the expression for the optimal power output and the corresponding thermal efficiency. The effect of each of the parameters, i.e. the operating temperatures, effectivenesses of each heat exchanger, heat capacitance rate of heat source/sink external fluids on the heat transfer to and from the heat engine, regenerative heat transfer, working fluid temperatures, power output and thermal efficiencies, for both cycles have been studied. Discussion of the numerical results is given in the end.

2. System description

It is well known that the working substance of Ericsson and Stirling cycles may be a gas, a magnetic material, etc., and for different substances, these cycles will have different performance characteristics. When the working substance of these cycles is an ideal gas, the Ericsson cycle consists of two isothermal and two isobaric processes, while the Stirling cycle consists of two isothermal and two isochoric processes, as shown in Fig. 1(a) with their T–s diagrams in Fig. 1(b) and (c). These cycles approximate the compression stroke of real engines as an isothermal process 1-2, with an irreversible heat rejection at constant temperature T_C. The heat addition to the working fluid from the regenerator is modelled as isobaric (in the

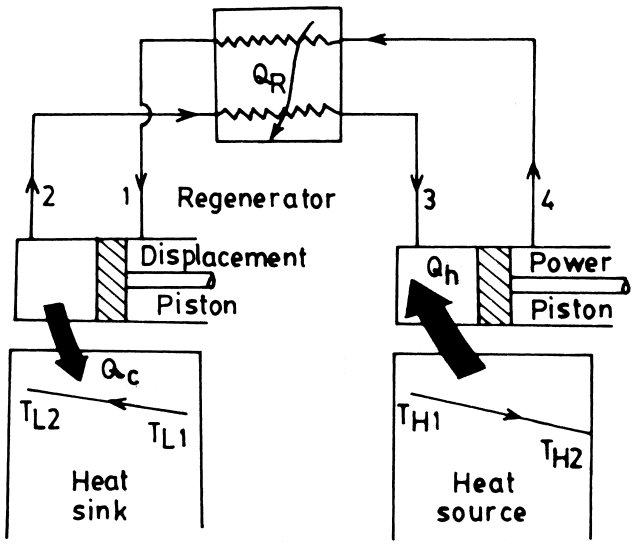


Fig. 1. (a) Schematic diagram of the Ericsson/Stirling heat engine cycle. (b) T–s diagram of Ericsson heat engine cycle. (c) T–s diagram of Stirling heat engine cycle.

Ericsson cycle) or isochoric (in the Stirling cycle), process 2-3. The work producing expansion stroke is modelled as an isothermal process 3-4, with irreversible heat addition at constant temperature T_h. Similarly, the heat rejection to the regenerator process is modelled as isobaric (in the Ericsson cycle)/isochoric (in the Stirling cycle), process 4-1.

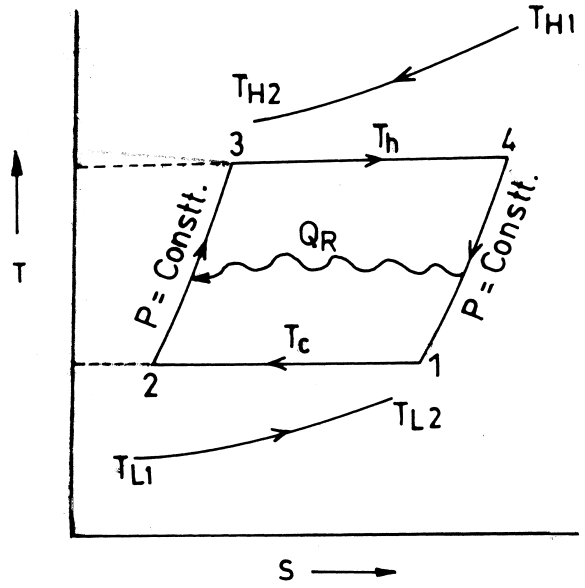
As mentioned earlier, the external heat transfer processes 1-2 and 3-4, within real Ericsson/Stirling heat engines must occur in finite time. This, in turn, requires that these heat transfer processes must proceed through finite temperature differences and are, therefore, defined as being externally irreversible. During the heat rejection process 1-2, heat flows from the working fluid, which is maintained at constant temperature T_C, to the low temperature heat sink of finite heat capacity whose temperature varies from T_{L1} to T_{L2}. Similarly, during the heat addition process 3-4, the heat flows from the high temperature heat source of finite heat capacity whose temperature varies from T_{H1} to T_{H2} to the working fluid, which is maintained at constant temperature T_h.

If the regenerator is an ideal one, the heat absorbed during process 4-1 is equal to the heat rejected during process 2-3, but an ideal regenerator requires an infinite area or an infinite regeneration time to transfer a finite amount of heat, and hence, it is desirable to consider a real regenerator with some losses.

3. Finite time thermodynamic analysis

An irreversible Ericsson/Stirling heat engine is sketched in Fig. 1(a) with their T–s diagrams in Fig. 1(b) and (c), respectively. Let Q_h be the amount of heat absorbed at temperature T_h from the heat source of finite heat capacity whose temperature varies from T_{H1} to T_{H2} and Q_C

be the amount of heat released at temperature T_C to the heat sink of finite heat capacity whose temperature varies from T_{L1} to T_{L2}. Thus, Q_h and Q_C are



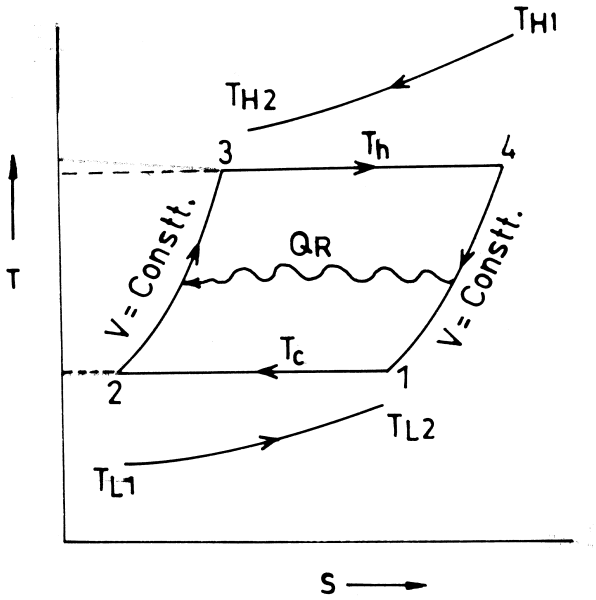


Fig. 1 (*continued*)

$$Q_h = T_h \Delta S \tag{1}$$

$$Q_C = T_C \Delta S \tag{2}$$

where ΔS is the entropy generation during the two isothermal processes, defined as:

$$\Delta S = nR \ln \lambda_f \tag{3}$$

where n is the number of moles of the working fluid, R is the universal gas constant and λ_f is the ratio of pressure/volume during the regenerative processes, i.e.

$\lambda_f = p_2 p_1$ for the Ericsson cycle and $p_2 > p_1$

$\lambda_f = v_1/v_2$ for the Stirling cycle and $v_1 > v_2$

Again, from heat transfer theory, the amounts of heat Q_h absorbed from the heat source and Q_C released to the heat sink are directly proportional to the Log Mean Temperature Difference (LMTD) as

$$Q_h = U_H A_H (\mathrm{LMTD})_H t_H \tag{4}$$

$$Q_C = U_L A_L (\mathrm{LMTD})_L t_L \tag{5}$$

where $U_H\ A_H$ and $U_L\ A_L$ are the overall heat transfer coefficient and area products

$$(\mathrm{LMTD})_H = \left[\frac{(T_{H1} - T_h) - (T_{H2} - T_h)}{\ln\left[(T_{H1} - T_h)/(T_{H2} - T_h)\right]} \right] \quad \text{and} \quad (\mathrm{LMTD})_L = \left[\frac{(T_C - T_{L1}) - (T_C - T_{L2})}{\ln\left[(T_C - T_{L1})/(T_C - T_{L2})\right]} \right]$$

Also, from the property of the external fluids

$$Q_h = C_H (T_{H1} - T_{H2}) t_H \tag{6}$$

$$Q_C = C_L (T_{L2} - T_{L1}) t_L \tag{7}$$

where C_H and C_L are the heat capacitance rates of the external fluids in the heat source and heat sink, respectively. From Eqs. (4)–(7), we have

$$Q_h = C_H \epsilon_H (T_{H1} - T_h) t_H \tag{8}$$

$$Q_C = C_L \epsilon_L (T_C - T_{L1}) t_L \tag{9}$$

where ϵ_H and ϵ_L are the effectivenesses of the high and low temperature heat exchangers, respectively, and are defined as

$$\epsilon_H = 1 - \exp(-N_H) \tag{10}$$

$$\epsilon_L = 1 - \exp(-N_L) \tag{11}$$

where N_H and N_L are the number of heat transfer units

$$N_H = U_H A_H / C_H \quad \text{and} \quad N_L = U_L A_L / C_L$$

It is also desirable to consider the influence of the irreversibility in the regenerator due to the finite heat transfer besides the two isothermal processes. Let ΔQ_R, be the heat loss during the two regeneration processes.

$$\Delta Q_R = n(1 - \epsilon_R) C_f (T_h - T_C) \tag{12}$$

where C_f is the specific heat of the working fluid during the two regenerative processes, i.e. $C_f = C_p$ for the Ericsson cycle and $C_f = C_v$ for the Stirling cycle.

Owing to the influence of irreversibility of the finite heat transfer, the regenerative processes time is not negligible in comparison to that of the two isothermal processes. In order to calculate the regenerative processes time, there are several methods and one of them is given [20] as

$$t_R = t_3 + t_4 = \pm \frac{2}{U}(T_h - T_C) \tag{13}$$

where U is the proportionality constant which is independent of the temperature difference and dependent only on the property of the regenerative material, and the $\pm$ sign belongs to the heating and cooling processes, respectively. Thus, the total time t_{cycle} is given by

$$t_{cycle} = t_H + t_L + t_R \tag{14}$$

In addition, there is a heat leak loss directly from the source to the sink which is directly proportional to the temperature difference and the cycle time, i.e.

$$Q_0 = k_0 (T_{HM} - T_{LM}) t_{cycle} \tag{15}$$

where k_0 is the heat leak coefficient from heat source to heat sink, T_{HM} and T_{LM} are the average temperatures of the heat source and heat sink, respectively, and are defined as

$$T_{HM} = (T_{H1} + T_{H2})/2 \quad \text{and} \quad T_{LM} = (T_{L1} + T_{L2})/2 \tag{16}$$

From Eqs. (8)–(11), we have

$$T_{H2} = (1 - \epsilon_H) T_{H1} + \epsilon_H T_h \quad \text{and} \quad T_{L2} = (1 - \epsilon_L) T_{L1} + \epsilon_L T_C \tag{17}$$

Thus, using Eqs. (15)–(17), Q_0 becomes

$$Q_0 = k_{01} + k_{02}(\epsilon_H T_h - \epsilon_L T_C) \tag{18}$$

where

$$k_{01} = k_0 \left[(2 - \epsilon_H) T_{H1} - (2 - \epsilon_L) T_{L1} \right] / 2 \quad \text{and} \quad k_{02} = k_0/2 \tag{18a}$$

When all three major irreversibilities mentioned above are taken into account, the net heat released from the heat source (Q_H) and absorbed by the heat sink (Q_L) are, respectively, given below:

$$Q_H = Q_h + \Delta Q_R + Q_0 \tag{19}$$

$$Q_L = Q_C + \Delta Q_R + Q_0 \tag{20}$$

The power output and the corresponding thermal efficiency are given by

$$P = \frac{W}{t_{cycle}} = \frac{(Q_H - Q_C)}{t_h + t_L + t_R} = \left[\frac{(T_h - T_C)}{\frac{T_h}{k_1(T_{H1} - T_h)} + \frac{T_C}{k_2(T_C - T_{L1})} + b(T_h - T_C)} \right] \tag{21}$$

and

$$\eta = \left[\frac{(T_h - T_C)}{T_h + a(T_h - T_C) + \left[k_{01} + k_{02}(\epsilon_H T_h - \epsilon_L T_C)\right]\left[\frac{T_h}{k_1(T_{H1} - T_h)} + \frac{T_C}{k_2(T_C - T_{L1})} + b(T_h - T_C)\right]} \right] \tag{22}$$

where

$k_1 = \epsilon_H C_H$, $k_2 = \epsilon_L C_L$, $a = C_f(l - \epsilon_H)/R \ln \lambda_f$ and $b = 2U/nR \ln \lambda_f$

From Eqs. (21) and (22), we see that the output power and the corresponding thermal efficiency are functions of two variables T_h and T_C. Thus, optimising P w. r. t. T_h and T_C, viz.

$$\frac{\partial P}{\partial T_h} = 0 \quad \text{and} \quad \frac{\partial P}{\partial T_C} = 0$$

we get,

$$T_h = \left[\frac{T_{H1}\sqrt{k_1/k_2} + \sqrt{T_{H1} T_{L1}}}{1 + \sqrt{k_1/k_2}} \right] \quad \text{and} \quad T_C = \left[\frac{T_{L1} + \sqrt{T_{H1} T_{L1} k_1/k_2}}{1 + \sqrt{k_1/k_2}} \right] \tag{23}$$

Substituting the values of T_h and T_C into Eqs. (21) and (22), we have the maximum power output and the corresponding thermal efficiency as below:

$$P_m = \frac{K\left[\sqrt{T_{H1}} - \sqrt{T_{L1}}\right]^2}{1 + bK\left[\sqrt{T_{H1}} - \sqrt{T_{L1}}\right]^2} \tag{24}$$

$$\eta_m = \left[\frac{\eta_{CA}}{\left(1 + a\eta_{CA}\right) + \frac{(k_{01} + k_{02}k_{03})\left(1 + bKT_{H1}\eta_{CA}^2\right)}{KT_{H1}\eta_{CA}}} \right] \tag{25}$$

where

$$K = \frac{k_1 k_2}{\left(\sqrt{k_1} + \sqrt{k_2}\right)^2}$$

$$k_{03} = \left[\left(\sqrt{\frac{k_1 T_{H1}}{k_2}} + \sqrt{T_{L1}} \right) \frac{\epsilon_H \sqrt{T_{H1}} - \epsilon_L \sqrt{T_{L1}}}{1 + \sqrt{k_1/k_2}} \right]$$

$\eta_{CA} = 1 - \sqrt{T_{L1}/T_{H1}}$ is the Curzon–Ahlborn efficiency of an endoreversible Carnot heat engine.

It is seen from Eq. (22) that the maximum power output of Ericsson and Stirling heat engines is not affected by the regenerative heat losses and the direct heat leak losses, although it depends on the time of the regenerative processes. It can also be proved that when the time of the two adiabatic processes in an endoreversible Carnot heat engine is given by Eq. (13) with the direct heat leak loss given by Eq. (15), the optimum power output is also given by Eq. (24), although it is seen from Eq. (25) that the thermal efficiency of the Ericsson/Stirling heat engine is different from that of an endoreversible Carnot heat engine and always lower than it for the same operating conditions (i.e. $\eta_m < \eta_{CA}$). We further consider some other special cases:

1. When $\epsilon_R = 1.00$, the Ericsson/Stirling engine possesses the condition of perfect regeneration, although the regenerative time is still considered. The maximum power efficiency is given by

$$\eta_m = \left[\frac{\eta_{CA}}{1 + \frac{(k_{01} + k_{02} k_{03})\left(1 + bKT_{H1}\eta_{CA}^2\right)}{KT_{H1}\eta_{CA}}} \right] \tag{26}$$

   In this case, not only the maximum power but also the corresponding thermal efficiency is given by the same expression for the Ericsson/Stirling and the endoreversible Carnot heat engines operating in the same temperature range and having the same heat leak loss described by Eq. (15).
2. When $k_0 = 0.0$, i.e. there is no direct heat leak from the heat source to the heat sink, the maximum power efficiency of the Ericsson/Stirling heat engine is given by

$$\eta_m = \frac{\eta_{CA}}{\left(1 + a\eta_{CA}\right)} \tag{27}$$

   which is different from that of an endoreversible Carnot heat engine without heat leak loss. It is similar to that which has been discussed by earlier authors [19].
3. When $t_R \propto (t_H + t_L)$, i.e. the time of the two regeneration processes is directly proportional to the two isothermal processes time or

$$t_R = \gamma(t_H + t_L) \tag{28}$$

   where γ is the proportionality constant, the maximum power output and the corresponding thermal efficiency are given by

$$P_m = \frac{K\left[\sqrt{T_{H1}} - \sqrt{T_{L1}}\right]^2}{1 + \gamma} \tag{29}$$

$$\eta_m = \left[\frac{\eta_{CA}}{\left(1 + a\eta_{CA}\right) + \frac{(k_{01} + K_{02} k_{03})(1+\gamma)}{KT_{H1}\eta_{CA}}} \right] \tag{30}$$

The maximum power output is identical to that of an endoreversible Carnot heat engine in which the time of the two adiabatic processes is given by Eq. (28), while the corresponding thermal efficiency is lower than the Curzon–Ahlborn efficiency.

4. When $t_R = 0.0$, although the regenerative losses are still considered, the maximum power output and the corresponding thermal efficiency are given by

$$P_m = K\left[\sqrt{T_{H1}} - \sqrt{T_{L1}}\right]^2 \tag{31}$$

$$\eta_m = \left[\frac{\eta_{CA}}{(1 + a\eta_{CA}) + \frac{(k_{01} + K_{02}k_{03})}{KT_{H1}\eta_{CA}}}\right] \tag{32}$$

In this case, the maximum power output is the same as that of an endoreversible Carnot heat engine, while the corresponding thermal efficiency is still lower than the Curzon–Ahlborn efficiency.

5. When $k_0 = t_R = 0.0$ and $\epsilon_R = 1.00$, the Ericsson/Stirling heat engine possesses the condition of perfect regeneration and the time of the two regeneration processes is negligible. Then the maximum power output is still given by Eq. (31) and the corresponding thermal efficiency is given by

$$\eta_m = \eta_{CA} \tag{33}$$

Thus, in this case, the Ericsson/Stirling heat engine is identical to that of an endoreversible Carnot heat engine operating at the same boundary conditions.

4. Numerical results and discussion

In order to have a numerical appreciation of the results, we have considered the inlet source and sink temperatures as $T_{H1} = 1300$ K and $T_{L1} = 300$ K, the effectiveness of each of the heat exchangers in the range from 0.40 to 1.00, and the heat capacitance rates of the external fluids in the range 0.30–1.80 kW/K. We have studied the effect of each of the parameters (while keeping the others as constant) on the heat transfer to and from the heat engine (Q_H and Q_L), the regenerative heat transfer (Q_R), the working fluid temperatures (T_h and T_C), the maximum power output (P_m) and the thermal efficiencies of both (Ericsson and Stirling) cycles, and the discussion of results is given below:

Tables 1(a–c) and 2(a–c) show the effect of the effectivenesses of the hot, cold and regenerative side heat exchangers (ϵ_H, ϵ_L and ϵ_R) on the various heat transfers, working fluid temperatures, maximum power output and thermal efficiencies of the Ericsson and Stirling heat engines, respectively.

4.1. Effect of ϵ_H

It is seen from Tables 1(a) and 2(a) that as the effectiveness of the hot side heat exchanger

(ϵ_H) increases, the heat transfers to and from the heat engine (Q_H and Q_L), the regenerative heat transfer (Q_R), the working fluid temperatures (T_h and T_C), the maximum power output (P_m) and the corresponding thermal efficiencies (η_I and η_{II}) increase. The effect of ϵ_H is more pronounced for the maximum power output (P_m) and less pronounced for the thermal efficiency (η_I) for the Ericsson and Stirling engines.

4.2. Effect of ϵ_L

Tables 1(b) and 2(b) show that as the effectiveness of the cold side heat exchanger (ϵ_L) increases, the heat transfers to and from the heat engine (Q_H and Q_L), the regenerative heat transfer (Q_R) and the working fluid temperatures (T_h and T_C) decrease while the maximum

Table 1(a–c)
Effect of ϵ_H, ϵ_L and ϵ_R on the heat transfer, power output and the thermal efficiency of the Ericsson heat engine

ϵ_H	Q_H	Q_L	Q_{R1}	P_m	T_h	T_C	η_I	η_{II}
0.40	301.46	170.54	425.86	48.07	904.30	434.41	43.07	55.99
0.50	307.44	173.85	434.55	54.60	922.75	443.27	43.16	56.11
0.60	312.37	176.57	441.73	60.31	938.00	450.60	43.22	56.19
0.70	316.54	178.86	447.84	65.39	950.98	456.84	43.27	56.26
0.80	320.15	180.84	453.15	69.97	962.25	462.25	43.31	56.31
0.90	323.31	182.57	457.83	74.13	972.19	467.03	43.34	56.35
1.00	326.12	184.09	462.01	77.95	981.07	471.29	43.37	56.38
ϵ_L	Q_H	Q_L	Q_{R1}	P_m	T_h	T_C	η_I	η_{II}
0.40	339.23	191.54	480.44	48.07	1020.20	490.09	43.08	56.00
0.50	333.15	188.12	471.75	54.60	1001.75	481.23	43.17	56.12
0.60	328.12	185.31	464.57	60.31	986.50	473.90	43.23	56.20
0.70	323.85	182.91	458.46	65.39	973.52	467.66	43.28	56.26
0.80	320.15	180.84	453.15	69.97	962.25	462.25	43.31	56.31
0.90	316.88	179.01	448.47	74.13	952.31	457.47	43.34	56.34
1.00	313.97	177.39	444.29	77.95	943.43	453.21	43.37	56.38
ϵ_R	Q_H	Q_L	Q_{R1}	P_m	T_h	T_C	η_I	η_{II}
0.40	571.90	432.59	201.40	69.97	962.25	462.25	24.30	31.58
0.50	521.55	382.24	251.75	69.97	962.25	462.25	26.63	34.62
0.60	471.20	331.89	302.10	69.97	962.25	462.25	29.47	38.31
0.70	420.85	281.54	352.45	69.97	962.25	462.25	32.99	42.88
0.80	370.50	231.19	402.80	69.97	962.25	462.25	37.45	48.68
0.90	320.15	180.84	453.15	69.97	962.25	462.25	43.31	56.31
0.95	294.97	155.66	478.32	69.97	962.25	462.25	46.99	61.09
1.00	269.80	130.49	503.50	69.97	962.25	462.25	51.35	66.76

power output (P_m) and the corresponding thermal efficiencies (η_I and η_{II}) increase. The effect of ϵ_L is more pronounced for the maximum power output (P_m) and less pronounced for the thermal efficiency (η_I) for the Ericsson and Stirling engines.

4.3. Effect of ϵ_R

Tables 1(c) and 2(c) show that as the effectiveness of the regenerative heat exchanger (ϵ_R) increases, the heat transfers to and from the heat engine (Q_H and Q_L) decrease, the regenerative heat transfer (Q_R) and the thermal efficiencies (η_I and η_{II}) increase, while the working fluid temperatures (T_h and T_C) and the maximum power output (P_m) remain constant. The effect of ϵ_R is more pronounced for the maximum heat transfer from the heat engine (Q_L) and less pronounced for the thermal efficiency (η_I) for the Ericsson and Stirling engines.

Tables 1(d–e) and 2(d–e) show the effect of external fluids temperatures of the hot and cold side heat reservoirs (T_{H1} and T_{L1}) on the various heat transfers, working fluid temperatures, maximum power output and the corresponding thermal efficiencies of the Ericsson and Stirling heat engines, respectively.

4.4. Effect of T_{H1}

Tables 1(d) and 2(d) show that as the inlet temperature of the heat source increases, the heat transfers to and from the heat engine (Q_H and Q_L), the regenerative heat transfer (Q_R), the

Table 1(d–e)
Effect of T_{H1} and T_{L1} on the heat transfer, power output and the thermal efficiency of the Ericsson heat engine

T_{H1}	Q_H	Q_L	Q_{R1}	P_m	T_h	T_C	η_I	η_{II}
900	228.99	145.41	271.89	32.10	709.81	409.81	36.27	54.40
1000	252.04	154.53	317.20	40.83	773.86	423.86	38.46	54.95
1100	274.90	163.46	362.52	50.10	837.23	437.23	40.32	55.44
1200	297.60	172.22	407.83	59.83	900.00	450.00	41.92	55.89
1300	320.15	180.84	453.15	69.97	962.25	462.25	43.31	56.31
1400	342.57	189.33	498.47	80.46	1024.04	474.04	44.54	56.69
1500	364.87	197.70	543.78	91.27	1085.41	485.41	45.63	57.04
T_{L1}	Q_H	Q_L	Q_{R1}	P_m	T_h	T_C	η_I	η_{II}
290	319.20	178.50	457.68	72.15	957.00	452.00	43.88	56.48
295	319.68	179.67	455.42	71.05	959.64	457.14	43.60	56.39
300	320.15	180.84	453.15	69.97	962.25	462.25	43.31	56.31
305	320.61	182.00	450.88	68.90	964.84	467.34	43.03	56.22
310	321.06	183.15	448.62	67.85	967.41	472.41	42.75	56.14
315	321.51	184.30	446.35	66.82	969.96	477.46	42.47	56.05
320	321.96	185.44	444.09	65.80	972.49	482.49	42.20	55.97
325	322.40	186.57	441.82	64.80	975.00	487.50	41.92	55.89

working fluid temperatures (T_h and T_C), the maximum power output (P_m) and the corresponding thermal efficiencies (η_I and η_{II}) increase. The effect of T_{H1} is more pronounced for the maximum power output (P_m) and less pronounced for the thermal efficiency (η_I). Thus, it is desirable to have a high temperature heat source from the point of view of larger power output as well as the higher thermal efficiency for the Ericsson and Stirling heat engines.

Table 1(f–g)
Effect of C_H and C_L on the heat transfer, power output and the thermal efficiency of the Ericsson heat engine

C_H	Q_H	Q_L	Q_{R1}	P_m	T_h	T_C	η_I	η_{II}
0.30	287.43	162.41	406.67	35.11	863.55	414.84	42.91	55.79
0.40	294.94	166.64	417.34	42.06	886.21	425.72	43.04	55.96
0.50	300.94	170.02	425.86	48.07	904.30	434.41	43.13	56.07
0.60	305.93	172.83	432.95	53.36	919.35	441.64	43.19	56.14
0.70	310.19	175.23	439.00	58.11	932.21	447.82	43.23	56.20
0.80	313.91	177.33	444.29	62.41	943.43	453.21	43.26	56.24
0.90	317.20	179.18	448.96	66.34	953.36	457.98	43.29	56.28
1.00	320.15	180.84	453.15	69.97	962.25	462.25	43.31	56.31
1.10	322.81	182.34	456.94	73.33	970.30	466.12	43.33	56.33
1.20	325.25	183.71	460.39	76.46	977.63	469.64	43.35	56.35
1.30	327.48	184.97	463.57	79.39	984.37	472.88	43.36	56.37
1.40	329.54	186.13	466.50	82.15	990.59	475.87	43.37	56.39
1.50	331.46	187.21	469.22	84.76	996.37	478.64	43.38	56.40
1.60	333.24	188.21	471.75	87.22	1001.75	481.23	43.39	56.41
1.70	334.91	189.15	474.13	89.55	1006.79	483.65	43.40	56.42
1.80	336.48	190.04	476.36	91.77	1011.53	485.92	43.41	56.43
C_L	Q_H	Q_L	Q_{R1}	P_m	T_h	T_C	η_I	η_{II}
0.30	352.86	199.26	499.63	35.11	1060.95	509.66	42.87	55.73
0.40	345.35	195.04	488.96	42.06	1038.29	498.78	43.02	55.92
0.50	339.35	191.66	480.44	48.07	1020.20	490.09	43.11	56.04
0.60	334.37	188.85	473.35	53.36	1005.15	482.86	43.17	56.13
0.70	330.10	186.45	467.30	58.11	992.29	476.68	43.22	56.19
0.80	326.38	184.35	462.01	62.41	981.07	471.29	43.26	56.24
0.90	323.09	182.50	457.34	66.34	971.14	466.52	43.29	56.27
1.00	320.15	180.84	453.15	69.97	962.25	462.25	43.31	56.31
1.10	317.48	179.34	449.36	73.33	954.20	458.38	43.33	56.33
1.20	315.05	177.97	445.91	76.46	946.87	454.86	43.35	56.36
1.30	312.81	176.71	442.73	79.39	940.13	451.62	43.37	56.38
1.40	310.75	175.55	439.80	82.15	933.91	448.63	43.38	56.39
1.50	308.84	174.47	437.08	84.76	928.13	445.86	43.39	56.41
1.60	307.05	173.46	434.55	87.22	922.75	443.27	43.40	56.42
1.70	305.38	172.52	432.17	89.55	917.71	440.85	43.41	56.43
1.80	303.81	171.64	429.94	91.77	912.97	438.58	43.42	56.44

Table 2(a–c)
Effect of ϵ_H, ϵ_L and ϵ_R on the heat transfer, power output and the thermal efficiency of the Stirling heat engine

ϵ_H	Q_H	Q_L	Q_{R1}	P_m	T_h	T_C	η_I	η_{II}
0.40	273.83	150.26	304.49	48.06	904.30	434.41	44.76	58.18
0.50	279.25	153.16	310.70	54.59	922.75	443.27	44.86	58.31
0.60	283.71	155.54	315.83	60.30	938.00	450.60	44.93	58.40
0.70	287.49	157.54	320.20	65.38	950.98	456.84	44.98	58.47
0.80	290.75	159.26	324.00	69.95	962.25	462.25	45.02	58.53
0.90	293.61	160.76	327.35	74.11	972.19	467.03	45.06	58.57
1.00	296.15	162.09	330.34	77.93	981.07	471.29	45.08	58.61
ϵ_L	Q_H	Q_L	Q_{R1}	P_m	T_h	T_C	η_I	η_{II}
0.40	308.07	168.66	343.51	48.06	1020.20	490.09	44.77	58.20
0.50	302.55	165.66	337.30	54.59	1001.75	481.23	44.87	58.32
0.60	297.98	163.18	332.17	60.30	986.50	473.90	44.93	58.41
0.70	294.11	161.08	327.80	65.38	973.52	467.66	44.98	58.48
0.80	290.75	159.26	324.00	69.95	962.25	462.25	45.02	58.53
0.90	287.79	157.66	320.65	74.11	952.31	457.47	45.05	58.57
1.00	285.14	156.23	317.66	77.93	943.43	453.21	45.08	58.60
ϵ_R	Q_H	Q_L	Q_{R1}	P_m	T_h	T_C	η_I	η_{II}
0.40	470.75	339.26	144.00	69.95	962.25	462.25	27.85	36.21
0.50	434.75	303.26	180.00	69.95	962.25	462.25	30.15	39.20
0.60	398.75	267.26	216.00	69.95	962.25	462.25	32.87	42.73
0.70	362.75	231.26	252.00	69.95	962.25	462.25	36.12	46.95
0.80	326.75	195.26	288.00	69.95	962.25	462.25	40.08	52.11
0.90	290.75	159.26	324.00	69.95	962.25	462.25	45.02	58.53
0.95	272.75	141.26	342.00	69.95	962.25	462.25	47.98	62.37
1.00	254.75	123.26	360.00	69.95	962.25	462.25	51.35	66.76

4.5. Effect of T_{L1}

Tables 1(e) and 2(e) show that as the inlet temperature of the heat sink increases, the heat transfers to and from the heat engine (Q_H and Q_L) and the working fluid temperatures (T_h and T_C) increase, while the regenerative heat transfer (Q_R), the maximum power output (P_m) and the corresponding thermal efficiencies (η_I and η_{II}) decrease. The effect of T_{L1} is more pronounced for the cold side working fluid temperature (T_C) and less pronounced for the heat transfer to, the heat engine (Q_H). Thus, it is desirable to have a low temperature heat sink from the point of view of less heat input as well as higher power output for the Ericsson and Stirling heat engines.

Tables 1(f–g) and 2(f–g) show the effect of heat capacitance rates of external fluids in the hot and cold side heat reservoirs (C_H and C_L) on the various heat transfers, working fluid temperatures, maximum power output and the corresponding thermal efficiencies of the Ericsson and Stirling heat engines, respectively.

4.6. Effect of C_H

Tables 1(f) and 2(f) show that as the heat capacitance rates of the heat source external fluid (C_H) increase, the heat transfers to and from the heat engine (Q_H and Q_L), the regenerative heat transfer (Q_R), the working fluid temperatures (T_h and T_C), the maximum power output (P_m) and the corresponding thermal efficiencies (η_I and η_{II}) increase. The effect of C_H is more pronounced for the maximum power output (P_m) and less pronounced for the thermal efficiency (η_l) for the Ericsson and Stirling heat engines.

4.7. Effect of C_L

Tables 1(g) and 2(g) show that as the heat capacitance rates of the heat sink external fluid (C_L) increase, the heat transfers to and from the heat engine (Q_H and Q_L), the regenerative heat transfer (Q_R), the working fluid temperatures (T_h and T_C) decrease, while the maximum power output (P_m) and the corresponding thermal efficiencies (η_I and η_{II}) increase. The effect of C_L is more pronounced for the maximum power output (P_m) and less pronounced for the thermal efficiency (η_I). It is also seen from these tables that the heat capacitance rates of the heat sink fluid should be more than that of the heat source external fluid (i.e. $C_L > C_H$) from the point of view of higher power output (P_m) as well as for lower heat input (Q_H) to the heat engine and, hence, for lower inlet temperature of the heat source (T_{H1}) for the Ericsson and Stirling heat engines.

Table 2(d–e)
Effect of T_{H1} and T_{L1} on the heat transfer, power output and the thermal efficiency of the Stirling heat engine

T_{H1}	Q_H	Q_L	Q_{R1}	P_m	T_h	T_C	η_I	η_{II}
900	209.28	130.39	194.40	32.10	709.81	409.81	37.46	56.19
1000	229.90	137.86	226.80	40.83	773.86	423.86	39.80	56.86
1100	250.33	145.14	259.20	50.09	837.23	437.23	41.80	57.47
1200	270.61	152.27	291.60	59.82	900.00	450.00	43.52	58.02
1300	290.75	159.26	324.00	69.95	962.25	462.25	45.02	58.53
0140	310.77	166.13	356.40	80.44	1024.04	474.04	46.35	58.99
1500	330.68	172.89	388.80	91.25	1085.41	485.41	47.53	59.41
T_{L1}	Q_H	Q_L	Q_{R1}	P_m	T_h	T_C	η_I	η_{II}
290	289.74	156.94	327.24	72.13	957.00	452.00	45.64	58.74
295	290.25	158.10	325.62	71.03	959.64	457.14	45.33	58.63
300	290.75	159.26	324.00	69.95	962.25	462.25	45.02	58.53
305	291.24	160.41	322.38	68.89	964.84	467.34	44.72	58.42
310	291.73	161.56	320.76	67.84	967.41	472.41	44.41	58.32
315	292.21	162.70	319.14	66.80	969.96	477.46	44.11	58.22
320	292.69	163.83	317.52	65.79	972.49	482.49	43.82	58.12
325	293.16	164.96	315.90	64.79	975.00	487.50	43.52	58.02

Table 2(f–g)
Effect of C_H and C_L on the heat transfer, power output and the thermal efficiency of the Stirling heat engine

C_H	Q_H	Q_L	Q_{R1}	P_m	T_h	T_C	η_I	η_{II}
0.30	261.05	143.05	290.77	35.10	863.55	414.84	44.59	57.97
0.40	267.87	146.77	298.40	42.06	886.21	425.72	44.73	58.15
0.50	273.31	149.74	304.49	48.06	904.30	434.41	44.82	58.27
0.60	277.84	152.21	309.56	53.36	919.35	441.64	44.88	58.35
0.70	281.71	154.33	313.89	58.10	932.21	447.82	44.93	58.41
0.80	285.08	156.17	317.66	62.40	943.43	453.21	44.97	58.46
0.90	288.07	157.80	321.01	66.33	953.36	457.98	45.00	58.50
1.00	290.75	159.26	324.00	69.95	962.25	462.25	45.02	58.53
1.10	293.17	160.58	326.71	73.31	970.30	466.12	45.04	58.55
1.20	295.38	161.79	329.18	76.44	977.63	469.64	45.06	58.58
1.30	297.40	162.89	331.45	79.38	984.37	472.88	45.07	58.60
1.40	299.28	163.92	333.54	82.13	990.59	475.87	45.09	58.61
1.50	301.01	164.86	335.49	84.73	996.37	478.64	45.10	58.63
1.60	302.64	165.75	337.30	87.20	1001.75	481.23	45.11	58.64
1.70	304.15	166.58	339.00	89.53	1006.79	483.65	45.12	58.65
1.80	305.58	167.35	340.59	91.75	1011.53	485.92	45.13	58.67
C_L	Q_H	Q_L	Q_{R1}	P_m	T_h	T_C	η_I	η_{II}
0.30	320.45	175.47	357.23	35.10	1060.95	509.66	44.54	57.91
0.40	313.63	171.75	349.60	42.06	1038.29	498.78	44.70	58.11
0.50	308.19	168.78	343.51	48.06	1020.20	490.09	44.80	58.24
0.60	303.66	166.31	338.44	53.36	1005.15	482.86	44.87	58.33
0.70	299.79	164.19	334.11	58.10	992.29	476.68	44.92	58.40
0.80	296.41	162.35	330.34	62.40	981.07	471.29	44.96	58.45
0.90	293.42	160.72	326.99	66.33	971.14	466.52	45.00	58.49
1.00	290.75	159.26	324.00	69.95	962.25	462.25	45.02	58.53
1.10	288.33	157.94	321.29	73.31	954.20	458.38	45.04	58.56
1.20	286.12	156.73	318.82	76.44	946.87	454.86	45.06	58.58
1.30	284.09	155.63	316.55	79.38	940.13	451.62	45.08	58.60
1.40	282.22	154.60	314.46	82.13	933.91	448.63	45.09	58.62
1.50	280.48	153.66	312.51	84.73	928.13	445.86	45.11	58.64
1.60	278.86	152.77	310.70	87.20	922.75	443.27	45.12	58.65
1.70	277.34	151.94	309.00	89.53	917.71	440.85	45.13	58.67
1.80	275.92	151.17	307.41	91.75	912.97	438.58	45.14	58.68

5. Conclusions

Finite time thermodynamics has been applied to optimise the power output and the corresponding thermal efficiency of irreversible Ericsson and Stirling heat engines, including the finite heat capacitance rates of external fluids in the heat source and heat sink reservoirs, regenerative heat losses, direct heat leak losses, finite regeneration processes time and real hot and cold side heat exchangers (i.e. effectivenesses < 1.00). It is found that the effectivenesses of the hot and cold side heat exchangers (ϵ_H and ϵ_L), the source and sink inlet temperatures (T_{H1}

and T_{L1}) and the heat capacitance rates of the external fluids affect the maximum power output (P_m) and the thermal efficiency (η_I) while the regenerative effectiveness (ϵ_R) affects the thermal efficiency only. It can be seen that the heat source external inlet temperature (T_{H1}) should be high enough, while the heat sink inlet temperature should be low enough from the point of view of maximum power output, as well as the corresponding thermal efficiency. It is also found that the heat capacitance rate of the heat source should be lower than the heat capacitance rate of the heat sink (i.e. $C_H < C_L$) from the point of view of higher power output, as well as for lower heat input to the heat engine (Q_H), and hence, for lower inlet temperature of the heat source (T_{H1}). Hence, the present analysis provides a new theoretical basis for design, performance evaluation and improvements of both the Ericsson and Stirling heat engines, and the results are, moreover, the same for both engines.

References

[1] Curzon FL, Ahlborn B. Efficiency of Carnot cycle at maximum power output. American Journal of Physics 1975;43:22–4.
[2] Waker A. Stirling cycle machines. Oxford: Clarendon Press, 1993.
[3] Daub EE. The regenerator principle in the Stirling and Ericsson hot air engines. British Journal for the History of Science 1974;27(7 part 3):529–77.
[4] Blank DA, Wu C. Power limit of an endoreversible Ericsson cycle with regeneration. Energy Conversion & Management 1996;37:59–66.
[5] Marks AM. Electrothermodynamic (etd) power convertors. US Patent 4395648, 1983.
[6] Chen J, Yan Z. The effect of field dependent heat capacity on regeneration in magnetic Ericsson cycle. Journal of Applied Physics 1980;69:6245–7.
[7] Rao HV. Cry-refrigerator based on reversed Ericsson cycle. Indian Journal of Cryogenics 1980;5:46–51.
[8] Corey JA. A rotary air-cycle refrigerator with thermally active liquid pistons. Procs 25th Intersociety Energy Conversion Engineering Conference 1990;6:286–91.
[9] Murphy RW, Chen FC, Mei VC, Chen GL. Analysis of magnetic refrigeration cycles, Procs. 26th Intersociety Energy Conversion Engineering Conference 1991;2:482–6.
[10] Greenspan E. Liquid metal mist cooling MHD Ericsson cycles for fusion energy conversion. Procs IEEE 13th Symposium Fusion Energy 1990;2:891–7.
[11] Blumenau L, Branover H, Smith RJ. Liquid metal MHD power conversion for space electric systems. In: Nuclear Power Systems, Albuquerque, NM 12–16, January, 1987.
[12] Mills JL, Kirol LD, van Haaften DH. Magnetic heat pump cycles for industrial waste heat recovery. 9th Intersociety Energy Conversion Engineering Conference 1984;3:1369–74.
[13] Fieldhouse IB, Porter RW. Cryogenic cooling of infrared electronics. State of Art Review, May 1983 to May 1996, GACIAC, IIT Research Institute. Report No. GACIAC-SOAR-86-02, October, 1996.
[14] Elgendly YAM. Variable speed scroll Ericsson heat pump system. In: 13th Energy Engg. World Congress, Energy and Environmental Strategies for the 1990s, 1991. p. 609–16.
[15] Chen FC, Murphy RW, Chen GL, Mei VC. Loss analysis of the thermodynamic cycle magnetic of heat pumps. Oak Ridge TN National Lab., Report No. DE91-007669, ORNL/TM-i1608, 1991.
[16] Purnell JG. Performance predictions for room temperature Ericsson cycle magnetic heat pump. David Taylor Naval Research and Development Centre, Report No. AD-A114348, DTNRDC/PAS-82/11, 1982.
[17] Badescu V. Optimum operation of solar convertor in a combination with a Stirling or Ericsson heat engine. Energy 1992;17:601–7.
[18] Blank DA, Davis GW, Wu C. Performance limit of an endoreversible Stirling cycle with regeneration. In: 6th International Stirling Engine Conference, Rotterdam, The Netherlands, May 26–28, 1993.

[19] Blank DA, Wu C. Finite time power limit for solar-radiant Ericsson engines for space applications. Applied Thermal Engineering 1998;18:1347–57.
[20] Chen J, Schouten JA. The comprehensive influence of several major irreversibilities on the performance of an Ericsson heat engine. Applied Thermal Engineering 1999;19:555–64.
[21] Kaushik SC, Kumar S. Finite time thermodynamic analysis of endoreversibble Stirling heat engine with regenerative losses. Energy 2000, in press.